# Selected Solution to Test 1.

1. (a) $fl(x) = x(1+\delta)$, $|\delta| \le \varepsilon$

(b) $fl(x+y) = (x(1+\delta_1) + y(1+\delta_2))(1+\delta_3)$

$$= (x + x\delta_1 + y + y\delta_2) + (x+y)\delta_3 + O(\varepsilon^2)$$

$$= (x+y)\left(1 + \frac{x\delta_1 + y\delta_2}{x+y} + \delta_3\right) + O(\varepsilon^2)$$

$$= (x+y)(1+\bar{\delta})$$

$$|\delta| \le \frac{(|x|+|y|)\varepsilon}{|x+y|} + \varepsilon$$

Can not, no need to go further

$$fl(\sqrt{x}) = \sqrt{x(1+\delta_1)}\,(1+\delta_2) = \sqrt{x}\,(1+\delta_1)^{1/2}(1+\delta_2)$$

$$= \sqrt{x}\left(1 + \frac{\delta_1}{2} + \delta_2 + O(\varepsilon^2)\right)$$

$$= \sqrt{x}\,(1+\bar{\delta}), \quad |\delta| \le \frac{3}{2}\varepsilon.$$

(2) If $a<0$, use $\sqrt{a^2+\delta^2} - a$

$a>0$, $(\sqrt{a^2+\delta^2} - a)\cdot\dfrac{\sqrt{a^2+\delta^2}+a}{\sqrt{a^2+\delta^2}+a} = \dfrac{\delta^2}{\sqrt{a^2+\delta^2}+a}$

(3) $A = A^T$, $A^TA = A^2$, $\lambda_{max}(A^TA) = (\lambda_{max}(A))^2$

$\|A\|_2 = \max_i |\lambda_i(A)|$, $(\lambda-2)^2 - 1 = 0$, $\lambda - 2 = \pm 1$

$\|A\|_2 = 3$, $\|A\|_1 = \|A\|_2 = \|A\|_\infty = 3$

(4) (1) $\|I\| = \max_{\|x\|=1} \|Ix\| = \max_{\|x\|=1} \|x\| = 1$.

(2) $\underline{\|A\|} = \sup_{x\ne 0} \dfrac{\|Ax\|}{\|x\|} \ge \underline{\dfrac{\|Ax^*\|}{\|x^*\|}}$

$\Rightarrow \|Ax^*\| \le \|A\|\,\|x^*\|$

$$\|Qx\|_2^2 = (Qx)^T Qx = x^T Q^T Q x = x^T x = \|x\|_2^2$$

$$\Rightarrow \|Qx\|_2 = \|x\|_2$$

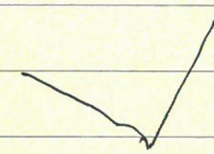
(5), Note that $A(:,3) = b$, the third column of $A$. Thus, $A e_3 = b$, the soln should be $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\det(A) = 24$.

(6) The error will decrease with $h$ until the round-off errors pick up. The best $h$ is the one that balances the formula error $O(h^2)$ and the round error, $O(\frac{\varepsilon}{h})$

Thus $\frac{\varepsilon}{h} \approx h^2 \Rightarrow h^3 = \varepsilon$, or $h = \sqrt[3]{\varepsilon}$.

when $\varepsilon = 10^{-16}$. $h = 10^{-16} \approx 10^{-5}$

as you saw from the HW1



$h \approx 10^{-5}$