MA/CSC 580 Review

1. Homework problems.
2. Questions and problems discussed in class.
3. Computer number systems and round-off error analysis, Wilkinson's backward error analysis. What is the machine precision?

   Find error bounds of $fl(x)$, $fl(x \circ y)$, $\circ = +, -, \times, \div\sqrt{x}$, etc. How about $fl(x \circ y \circ z)$?
4. How to avoid round-off errors? $f(x+h) - f(x)$, $1 - \cos x$, $b - \sqrt{b^2 - \delta}$, $f(x)/x$, ...
5. Definition of vector and matrix norms. Associate matrix norms. Cauchy-Schwartz inequality. Frobineuous norm.

   - Show that the $\|x\|_p$ norms are equivalent ($p = 1, 2, \infty$).
   - Show that the $\|A\|_p$ norms are equivalent ($p = 1, 2, \infty$).
   - Derive the expression for $\|A\|_p$ norms ($p = 1, 2, \infty$).
   - Show that $\|Qx\|_2 = \|x\|_2$; $\|QA\|_2 = \|A\|_2$; $\|Q\|_2 = 1$, where Q is a unitary matrix.
   - Show that $\|I\| = 1$; $\|Ax\| \le \|A\|\|x\|$, $\|AB\| \le \|A\|\|B\|$.
6. Given a matrix A. Find $A = LU$ or $PA = LU$, From $PA = LU$ factorization to solve $Ax = b$:

   - Compute Pb,
   - Forward substitution: $Ly = Pb$,
   - Backward substitution: $Ux = y$.
7. Suppose

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 \\ -2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   (a) Can L_1 or L_3 be a Gauss transformation matrix with partial pivoting? Why?

   (b) Compute L_1^{-1}, L_3^{-1}, $L_1 L_3$, and $L_1^{-1} L_3^{-1}$.

   (c) Compute P^{-1}, P^T, P^2, PL_3, and PL_3P.