GE $Ax=b$. $A^{(1)} = [A \,|\, b] \to A^{(2)} \cdots A^{(n-1)} = [U \,|\, \tilde{b}]$

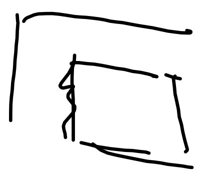
$k = 1, 2, \ldots, n-1$

$$a_{ij}^{(k+1)} = a_{ij}^{(k)} - \frac{a_{ik}^{(k)}}{a_{kk}^{(k)}} \cdot a_{kj}^{(k)}, \quad i = k, k+1, \ldots, n \quad j = k+1, \ldots, n, n+1$$

k-th step. $n-k$ divisions

multiplication $(n-k)(n-k)$

addition/subtraction $(n-k)(n-k+1)$



1-st. $(n-1)(n-1+1) + (n-1)$

$= (n-1)(n+1)$

2nd $(n-2)(n)$

3rd $(n-3)(n-1)$

$$\sum_{k=1}^{n-1} k(k+2) = \sum_{k=1}^{n-1} k^2 + \sum_{k=1}^{n-1} 2k$$

$$= \frac{(n-1)n(2n-1)}{6} + 2\,\frac{(1+n-1)}{2}(n-1)$$

handbook

$$= \left(\frac{1}{3}\right)n^3 + (\quad)^{n} + (\quad)$$

$$= O(n^3/3)$$

$n = 10^3$

$n^3 = 10^9$

320x320 ✓

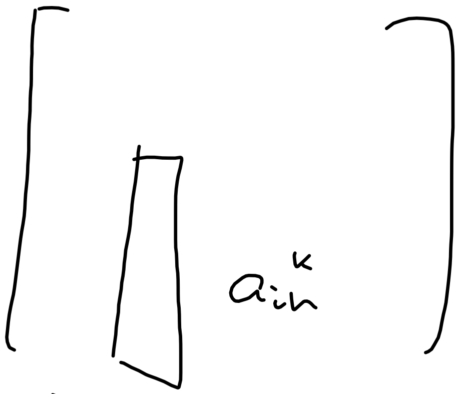
640x640 ✓

2480x2480 ✗

Storage. $A^{(1)} \to A^{(2)} \cdots, A^{(n-1)}$

Over-write.

$$\begin{bmatrix} a_{11}^{(1)} & \cdots & a_{1n}^{(1)} \\ \frac{a_{21}^{(1)}}{a_{11}^{(1)}} & a_{22}^{(2)} \cdots & a_{2n}^{(1)} \\ \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} & & \end{bmatrix}$$



for $k=1, n-1$

for $i=k:n$

$a_{ik} := \dfrac{a_{ik}}{a_{kk}}$ → L elements

for $j=k+1:n+1$

$a_{ij} := a_{ij} - a_{ik} * a_{kj};$ programming

end

end

end

When we finish, we get

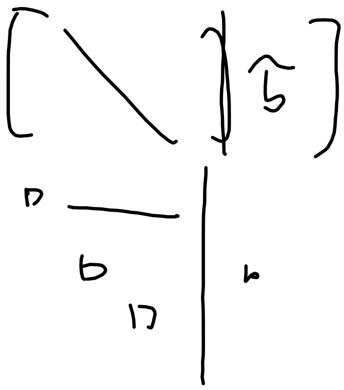
$A = LU$

1.
$$L = \begin{bmatrix} 1 & & & \\ a_{21} & 1 & & 0 \\ a_{31} & a_{32} & \ddots & \\ & & & \ddots \\ a_{n1} & \cdots & \cdots & a_{n,n-1} \end{bmatrix}$$

$$U = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & a_{22} & \cdots & a_{2n} \\ & & \ddots & \vdots \\ & & & a_{nn} \end{bmatrix}$$

2. Use backward substitution to solve

$Ax = b$ original problem



```
x_n = a_{n,n+1} / a_{nn}
for i = n-1, 1, -1
    x_i = a_{i,n+1}
    for j = i+1, n
        x_i = x_i - a_{ij} · x_j
    end
    x_i = x_i / a_{ii}
end
```

$x_i = a_{i,n+1} - \sum_{i} a_{ij} x$

(3) $\det(A) = a_{11} a_{22} \cdots a_{nn}$

a_{ii}'s are changed ones.

When will GE fail? $a_{ii} = 0$.

Does it mean $\det(A) = 0$, A is singular?

No.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad \det(A) = -1,$$

A^{-1} exist.

$$A = \begin{bmatrix} \sigma & 1 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} \sigma & 1 \\ \frac{1}{\sigma} & 1 - \frac{1}{\sigma} \end{bmatrix}$$

$$\overset{L}{\begin{bmatrix} 1 & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} \sigma & 1 \\ 0 & 1 - \frac{1}{\sigma} \end{bmatrix}} = \begin{bmatrix} \sigma & 1 \\ 1 & 1 \end{bmatrix} \checkmark$$

$$fl\left(\frac{1}{\sigma}\right) = \frac{1}{\sigma(1+\delta_1)}(1+\delta_2)$$

Switch rows does not change the problem.

Before GE, check

$$|a_{l1}| \geq |a_{i1}|, \quad i=1,2,\dots n.$$

```
l=1; pivot =abs(a(1,1));
for i=2:n
    if ( abs(a(i,1)) > pivot
        pivot = abs(a(i,1))
        l = i;
    end
end
```

$|a_{11}|$

$1 \to k$

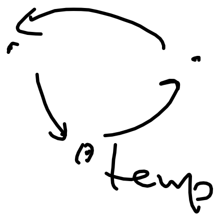
general case

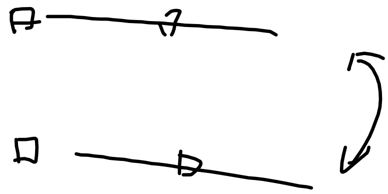
After finish, l is the index of row that has the largest magnitude.

$$pivot = \max |a_{i,1}|$$

Switch

```
for j=1: n+1
    tmp= a(1, j);
    a(1, j) = a(l, j);
    a(l, j) = tmp
end
```





$1 \longrightarrow k.$

Can be done using a vector operation

$b = a(:, j)$, j-th column of A.