List of mathematical functions

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In mathematics, a function or groups of functions are important enough to deserve their own names. This is a listing of articles which explain some of these functions in more detail. There is a large theory of special functions which developed out of statistics and mathematical physics. A modern, abstract point of view contrasts large function spaces, which are infinite-dimensional and within which most functions are 'anonymous', with special functions picked out by properties such as symmetry, or relationship to harmonic analysis and group representations.

See also List of types of functions

Contents

Elementary functions

Elementary functions are functions built from basic operations (e.g. addition, exponentials, logarithms...)

Algebraic functions

Algebraic functions are functions that can be expressed as the solution of a polynomial equation with integer coefficients.

- Polynomials: Can be generated by addition, multiplication, and exponentiation alone.
  - Constant function: polynomial of degree zero, graph is a horizontal straight line
  - Linear function: First degree polynomial, graph is a straight line.
  - Quadratic function: Second degree polynomial, graph is a parabola.
  - Cubic function: Third degree polynomial.
  - Quartic function: Fourth degree polynomial.
  - Quintic function: Fifth degree polynomial.

- - - Sextic function: Sixth degree polynomial.
  - Rational functions: A ratio of two polynomials.
  - *n*th root
    - Square root: Yields a number whose square is the given one.
    - Cube root: Yields a number whose cube is the given one.

Elementary transcendental functions

Transcendental functions are functions that are not algebraic.

- Exponential function: raises a fixed number to a variable power.
- Hyperbolic functions: formally similar to the trigonometric functions.
- Logarithms: the inverses of exponential functions; useful to solve equations involving exponentials.
  - Natural logarithm
  - Common logarithm
  - Binary logarithm
- Power functions: raise a variable number to a fixed power; also known as Allometric functions; note: if the power is a rational number it is not strictly a transcendental function.
- Periodic functions
  - Trigonometric functions: sine, cosine, tangent, cotangent, secant, cosecant, exsecant, excosecant, versine, coversine, vercosine, covercosine, haversine, hacoversine, havercosine, hacovercosine, etc.; used in geometry and to describe periodic phenomena. See also Gudermannian function.

Special functions

Basic special functions

- Indicator function: maps *x* to either 1 or 0, depending on whether or not *x* belongs to some subset.
- Step function: A finite linear combination of indicator functions of half-open intervals.
  - Heaviside step function: 0 for negative arguments and 1 for positive arguments. The integral of the Dirac delta function.
- Sawtooth wave
- Square wave
- Triangle wave
- Floor function: Largest integer less than or equal to a given number.
- Sign function: Returns only the sign of a number, as +1 or −1.
- Absolute value: distance to the origin (zero point)

Number theoretic functions

- Sigma function: Sums of powers of divisors of a given natural number.
- Euler's totient function: Number of numbers coprime to (and not bigger than) a given one.
- Prime-counting function: Number of primes less than or equal to a given number.
- Partition function: Order-independent count of ways to write a given positive integer as a sum of positive integers.

Antiderivatives of elementary functions

- Logarithmic integral function: Integral of the reciprocal of the logarithm, important in the prime number theorem.
- Exponential integral
- Trigonometric integral: Including Sine Integral and Cosine Integral

- Error function: An integral important for normal random variables.
    - Fresnel integral: related to the error function; used in optics.
    - Dawson function: occurs in probability.

Gamma and related functions

- Gamma function: A generalization of the factorial function.
- Barnes G-function
- Beta function: Corresponding binomial coefficient analogue.
- Digamma function, Polygamma function
- Incomplete beta function
- Incomplete gamma function
- K-function
- Multivariate gamma function: A generalization of the Gamma function useful in multivariate statistics.
- Student's t-distribution

Elliptic and related functions

- Elliptic integrals: Arising from the path length of ellipses; important in many applications. Related functions are the quarter period and the nome. Alternate notations include:
    - Carlson symmetric form
    - Legendre form
- Elliptic functions: The inverses of elliptic integrals; used to model double-periodic phenomena. Particular types are Weierstrass's elliptic functions and Jacobi's elliptic functions and the sine lemniscate and cosine lemniscate functions.
- Theta function
- Closely related are the modular forms, which include
    - J-invariant
    - Dedekind eta function

Bessel and related functions

- Airy function
- Bessel functions: Defined by a differential equation; useful in astronomy, electromagnetism, and mechanics.
- Bessel–Clifford function
- Legendre function: From the theory of spherical harmonics.
- Scorer's function
- Sinc function
- Hermite polynomials
- Laguerre polynomials
- Chebyshev polynomials

Riemann zeta and related functions

- Riemann zeta function: A special case of Dirichlet series.
- Riemann Xi function
- Dirichlet eta function: An allied function.
- Dirichlet L-function
- Hurwitz zeta function
- Legendre chi function
- Lerch transcendent
- Polylogarithm and related functions:
    - Incomplete polylogarithm

- Clausen function
      - Complete Fermi–Dirac integral, an alternate form of the polylogarithm.
      - Incomplete Fermi–Dirac integral
      - Kummer's function
      - Spence's function
  - Riesz function

Hypergeometric and related functions

- Hypergeometric functions: Versatile family of power series.
- Confluent hypergeometric function
- Associated Legendre functions
- Meijer G-function

Iterated exponential and related functions

- Hyper operators
- Iterated logarithm
- Pentation
- Super-logarithms
- Super-roots
- Tetration
- Lambert W function: Inverse of $f(w) = w \exp(w)$.

Other standard special functions

- Lambda function
- Lamé function
- Mittag-Leffler function
- Painlevé transcendents
- Parabolic cylinder function
- Synchrotron function

Miscellaneous functions

- Ackermann function: in the theory of computation, a computable function that is not primitive recursive.
- Dirac delta function: everywhere zero except for $x = 0$; total integral is 1. Not a function but a distribution, but sometimes informally referred to as a function, particularly by physicists and engineers.
- Dirichlet function: is an indicator function that matches 1 to rational numbers and 0 to irrationals. It is nowhere continuous.
- Thomae's function: is a function that is continuous at all irrational numbers and discontinuous at all rational numbers. It is also a modification of Dirichlet function and sometimes called Riemann function.
- Kronecker delta function: is a function of two variables, usually integers, which is 1 if they are equal, and 0 otherwise.
- Minkowski's question mark function: Derivatives vanish on the rationals.
- Weierstrass function: is an example of continuous function that is nowhere differentiable

See also

- List of mathematical abbreviations

External links

- Special functions (http://www.special-functions.com) : A programmable special functions calculator.
- Special functions (http://eqworld.ipmnet.ru/en/auxiliary/aux-specfunc.htm) at EqWorld: The World of Mathematical Equations.

Categories: Calculus │ Mathematics-related lists │ Number theory

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