MA 780: Test 1 Review Sheet

Note: The final is half open. You can write down anything on a piece of standard A4 paper.

1. Problems from the homework.

2. Questions asked at the beginning and in classes.

3. What is a polynomial interpolation? Is the problem well-posed? How many different methods are three (at lest three)? What is the best algorithm and its complexity? What are the advantages and limitations of Lagrange and Newton methods for polynomial interpolations?

4. Show that the Newton's polynomial interpolation is equivalent to the Lagrange one.

5. Show that

$$f[x_0, x_1, \cdots, x_n] = \sum_0^n \frac{f(x_i)}{\omega'_{n+1}(x_i))}$$

.

$$f[x_0, x_1, \cdots, x_n, x] = \frac{f^{(n+1)}(\xi)}{(n+1)!}.$$

6. How to use divided differences with multiple (such as double, triple etc.) nodes?

7. Why do we have Runge's phenomena of polynomial interpolations?

8. If we use a polynomial interpolation to approximate a function $f(x)$, can we get an error estimate and prove it?

9. Which is better, a polynomial interpolation or a piecewise polynomial interpolation? Try to explore different scenarios.

10. Show that the Lagrange and Newton polynomial interpolation are equivalent.

11. Derive the Hermite interpolation and an error estimate.

12. What are the degree of freedom (DOF) of piecewise linear, quadratic, cubic functions in $C[x_0, x_n], C^1[x_0, x_n], C^2[x_0, x_n]$?

13. Do you know the error estimates of piecewise linear, quadratic, cubic functions interpolations? Consider $C[x_0, x_n], C^1[x_0, x_n], C^2[x_0, x_n]$.

14. Find a set of basis functions for piecewise linear, quadratic, cubic functions interpolations in $C[x_0, x_n], C^1[x_0, x_n], C^2[x_0, x_n]$ given a mesh $\{x_i\}_0^n$.

15. What is a spline? What is a B-spline?

16. List some properties of a cubic spline interpolation and prove them.

17. What are the moments of splines?

18. What are common boundary conditions for cubic splines? How to find the system of equations for the coefficients. Are the interpolation problems wellposed?

19. Derive the closed and open Newton-Cotes formulas and show the error estimates. Should we use even and odd number of points for the quadrature formulas? Are they stable? What the meaning of the stability?

20. Derive composite quadrature formulas and show the error estimates.

21. Derive the relation between composite trapezoidal and Simpson's composite quadrature formulas.

22. What is a Richardson extrapolation? What is a Romberg integration?

23. What is the algebraic precision of a quadrature formula? What's the best one point quadrature formula and what is the algebraic precision?

24. What are the differences of a prior and posterior error analysis?

25. Let the nodal points be $x_i = 0, 1, 4$, and the function values be $y_i = \sqrt{x}$.

    (a) Find the least squares approximation using $y = a_0 + a_1 x$. How is the solution defined?

    (b) Find *both* the Lagrange polynomial and Newton polynomial interpolations, $p_2(x)$. Show the two results are the same. Approximate $y(x)$ at $x = 3$ and find the error.

    (c) Give a least upper bound of the error $\max_{0 \le x \le 4} |\sqrt{x} - p_2(x)|$.

    (d) Use your result to find a quadrature formula for $\int_0^4 f(x)dx$ and give an error estimate.

26. Given $(t, f(t))$, $(t+h, f(t+h))$, $(t+2h, f(t+2h))$.

    (a) Find the polynomial interpolation to approximate $f(t)$.

    (b) Give an error estimate for the polynomial interpolation between the interval $[t, t+2h]$.

    (c) From the polynomial interpolation, derive a quadrature formula to approximate $\int_t^{t+2h} f(t)dt$ and give an error estimate.