

Lab Session, Mathematics 587

Purpose: Learn Matlab *Partial Differential Equation* toolbox. Use it to solve some typical second order partial differential equations on some regions with certain boundary conditions; Visualize the mesh triangulation and the solutions; Export the triangulation.

Reference: Partial Differential Equation Toolbox, MathWorks.

Test Problems

1. Poisson equation on a unit circle:

$$\begin{aligned} -\Delta u &= 1, & x^2 + y^2 < 1 \\ u|_{\partial\Omega} &= 0, & x \leq 0 \\ u_n|_{\partial\Omega} &= 1, & x > 0. \end{aligned}$$

2. Wave equation on a unit square $[-1 \ 1] \times [-1 \ 1]$:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \Delta u, \\ u(x, y, 0) &= \arctan\left(\cos \frac{\pi x}{2}\right) \\ u_t(x, y, 0) &= 3 \sin(\pi x) e^{\sin \frac{\pi y}{2}} \\ u = 0 \quad \text{on} \quad x = -1, \quad \text{and} \quad x = 1, \quad u_n = 0 \quad \text{on} \quad y = -1, \text{and} \quad y = 1. \end{aligned}$$

3. Eigenvalue problem on a L-shape.

$$-\Delta u = \lambda u, \quad u = 0 \quad \text{on} \quad \partial\Omega.$$

The domain is the L-shape with corners $(0,0)$, $(-1,0)$, $(-1,-1)$, $(1,-1)$, $(1,1)$, and $(0,1)$.

4. Heat equation: A heated metal block

$$\frac{\partial u}{\partial t} = \Delta u,$$

The domain is the rectangle $[-0.5 \ 0.5] \times [-0.8 \ 0.8]$ with a rectangle cavity $[-0.05 \ 0.05] \times [-0.4 \ 0.4]$. the boundary condition is the following:

- $u = 100$ on the left-hand side.
- $u = -10$ on the right-hand side.

- $u_n = 0$ on other boundaries.

5. Download the Matlab source code 2D.rar either from Moodle or class-web page.

Export the mesh of the last testing problem from Matlab and Run `assemb.m` to solve Example 9.16 in the book.

General Procedure

- Draw the geometry.
- Define the boundary conditions.
- Define the partial differential equation.
- Define the initial conditions if necessary.
- Solve the PDE.
- Plot the solution.
- Refine the mesh if necessary.
- Save and quit.