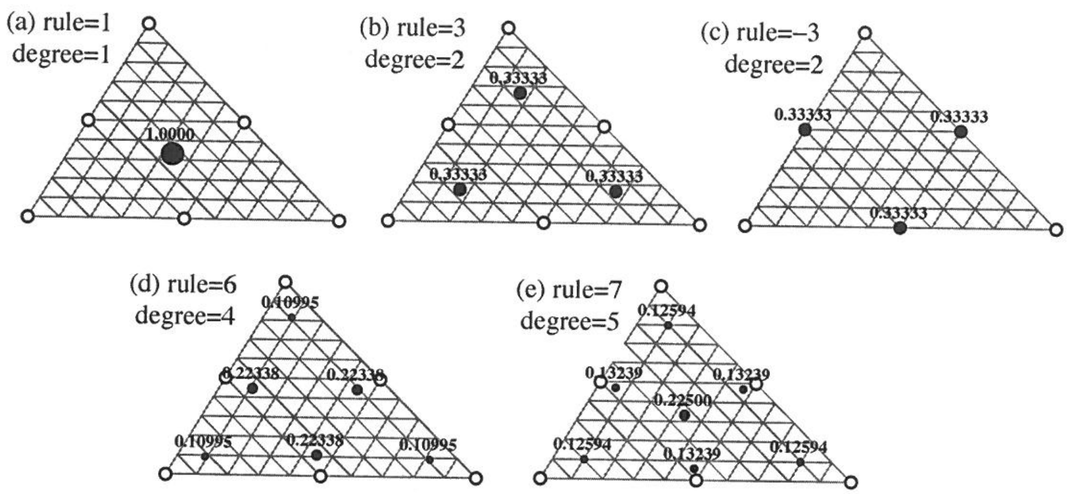

FIGURE 24.1. Location of sample points, marked as dark circles, of five Gauss quadrature rules for constant metric (a.k.a. superparametric) 6-node triangles (these have straight sides with side nodes located at the midpoints). Weight written to 5 places near each sample point; sample-point circle areas are proportional to weight.

Additional requirements for a Gauss rule to be numerically acceptable are:

> All sample points must be inside the triangle (or on the triangle boundary) and all weights must be positive. (24.2)

This is called a *positivity* condition. It insures that the element internal energy evaluated by numerical quadrature is nonnegative definite.

A rule is said to be of degree n if it integrates exactly all polynomials in the triangular coordinates of order n or less when the Jacobian determinant is constant, and there is at least one polynomial of order $n+1$ that is not exactly integrated by the rule.

Remark 24.2. The positivity requirement (24.2) is automatically satisfied in quadrilaterals by using Gauss product rules, since the points are always inside while all weights are positive. Consequently it was not necessary to call attention to it. On the other hand, for triangles there are Gauss rules with as few as 4 points that violate positivity.

§24.2.2. Superparametric Triangles

We first consider superparametric (constant metric) straight-sided triangles whose geometry is fully defined by the three corner nodes. Over such triangles the Jacobian determinant defined below is constant. The five simplest Gauss rules that satisfy the requirements (24.1) and (24.2) have $1, 3, 3,$ 6 and 7 points, respectively. The two rules with 3 points differ in the location of the sample points. The five rules are depicted in Figure 24.1 over 6-node straight-sided triangles; for such triangles to be superparametric the side nodes must be located at the midpoint of the sides.