Summary of Pi Theorem

$$u = f(w_1, w_2, \ldots, w_n)$$

$$= f(p_1, p_2, \ldots p_m, S_1, S_2, \ldots S_{n-m})$$

Let: $D = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_m^{\alpha_m}$, $[D] = [u]$

$$\Pi = \frac{u}{D} = \frac{1}{D} f(p_1, p_2, \ldots, p_m, S_1, S_2, \ldots S_{n-m})$$

Let $D_j = p_1^{\alpha_{j,1}} p_2^{\alpha_{j,2}} \ldots p_m^{\alpha_{j,m}}$, $[D_j] = [S_j]$

$$\Pi_j = S_j / D_j$$

dimensionless

$$\Pi = \frac{u}{D} = \frac{1}{D} f(p_1, p_2, \ldots p_m, D_1\Pi_1, D_2\Pi_2, \ldots D_{n-m}\Pi_{n-m})$$

$$= F(\Pi_1, \Pi_2, \ldots, \Pi_{n-m}) \quad \text{Different } F$$

$$\Rightarrow \quad u = D\, F(\Pi_1, \Pi_2, \ldots, \Pi_{n-m})$$

$$u = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_m^{\alpha_m} F\left(S_1 p_1^{-\alpha_{1,1}} p_2^{-\alpha_{1,2}} \cdots p_m^{-\alpha_{1,m}};\ S_2 p_1^{-\alpha_{2,1}} \cdots p_m^{-\alpha_{2,m}}, \ldots S_{n-m} p_1^{-\alpha_{n-m,1}} \cdots p_m^{-\alpha_{n-m,m}}\right)$$

Done !