1. $u = f(p_1, p_2, p_3, \theta)$ $\quad [\theta] = 1$ dimensionless

Then $D = p_1^{\alpha} p_2^{\beta} p_3^{\gamma}$, such that $[u] = [D]$ $\quad \pi = \frac{u}{D}$

$$u = D F(\theta) = p_1^{\alpha} p_2^{\beta} p_3^{\gamma} F(\theta)$$

2. $u = f(p_1, p_2, p_3, \theta_1, \theta_2)$ $\quad [\theta_1] = 1, \quad [\theta_2] = 1$

dimensionless

then $u = D F(\theta_1, \theta_2) = p_1^{\alpha} p_2^{\beta} p_3^{\gamma} F(\theta_1, \theta_2)$

3 $u = f(p_1, p_2, p_3, p_4, p_5, \theta_1, \theta_2)$ $\quad [\theta_1] = 1, \quad [\theta_2] = 1$

$D = p_1^{\alpha} p_2^{\beta} p_3^{\gamma}$ $\qquad D_1 = p_1^{\alpha_1} p_2^{\beta_1} p_3^{\gamma_1}$

such that $[D_1] = [p_4]$ $\qquad D_2 = p_1^{\alpha_2} p_2^{\beta_2} p_3^{\gamma_2}$

$[D_2] = [p_5]$

Then

$$u = D F(\pi_4, \pi_5, \theta_1, \theta_2)$$

$$= D F\left( \frac{p_1^{\alpha_1} p_2^{\beta_1} p_3^{\gamma_1} p_4}{p_1^{\alpha} p_2^{\beta} p_3^{\gamma}}, \frac{p_1^{\alpha_2} p_2^{\beta_2} p_3^{\gamma_2} p_5}{p_1^{\alpha} p_2^{\beta} p_3^{\gamma}}, \theta_1, \theta_2 \right)$$

4. $u = f(p_1, p_2, p_3) \rightarrow u = p_1^{\alpha} p_2^{\beta} p_3^{\gamma} F$ (F: constant)

$$u = C p_1^{\alpha} p_2^{\beta} p_3^{\gamma}$$

$\gamma = 0$