Answer to Sample Quiz 1

1\. (a) $|AD| = \sqrt{(-1)^2 + 0^2 + 1^2} = \sqrt{2}$

(b) $\vec{AB} = \langle 1, -1, 0 \rangle, \quad \vec{AC} = \langle 0, 1, 1 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = -\vec{i} - \vec{j} + \vec{k}$$

$|\vec{AB} \times \vec{AC}| = \sqrt{3}$, The area $= \sqrt{3}$,

A, B, and C are NOT in a same line.

(c) $$\frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{1}{\sqrt{3}} \langle -1, -1, 1 \rangle \quad \text{or} \quad \left\langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

(d) $$\cos\theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{-1}{\sqrt{2}\sqrt{2}} = -\frac{1}{2}, \qquad \theta = 120^\circ$$

(e) Component Projection

$$\text{comp}_{\vec{AC}} \vec{AB} = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AC}|} = \frac{-1}{\sqrt{2}}$$

Vector projection $\text{proj}_{\vec{AC}} \vec{AB} = \text{comp}_{\vec{AC}} \vec{AB} \, \frac{\vec{AC}}{|\vec{AC}|}$

$$= -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle 0, 1, 1 \rangle$$

$$= \frac{1}{2} \langle 0, 1, 1 \rangle.$$

(f) $(x+1)^2 + y^2 + (z-1)^2 = 2^2$

(g) Parametric form, Symmetric form

$$\begin{cases} x = t \\ y = -t \\ z = 0 \end{cases} \qquad \frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$$

(h) $\left(\frac{1}{2}, -\frac{1}{2}, 0\right)$

(i) $\overrightarrow{AD} = \langle -1, 0, 1 \rangle$

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 1 + 1 + 0 = 2 \neq 0$$

A, B, C, and D are NOT co-planar.

The volume of the parallelepiped $= 2$

(j) The point $(0, 1, 1)$, the direction $\overrightarrow{AB} = \langle 1, -1, 0 \rangle$

The equation of the plane is

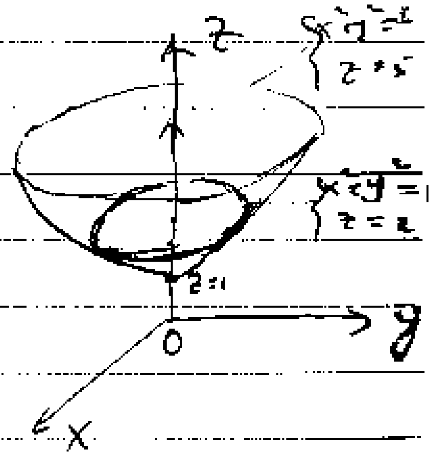
$$x - (y-1) = 0 \quad \text{or} \quad x - y + 1 = 0$$

(k) $$D = \frac{|E g|}{\sqrt{1^2 + (-1)^2 + 0^2}} = \frac{|-1 - (0-1)|}{\sqrt{2}} = 0$$

The point D is on the plane

2 (a) It is an elliptic paraboloid

$z - 1 = x^2 + y^2$



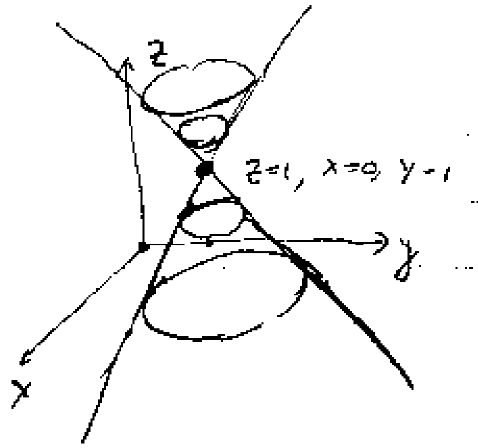
$z = 1, \qquad 0 = x^2 + y^2$

$z = 2 \qquad 1 = x^2 + y^2$

$z = k \qquad k - 1 = x^2 + y^2$ Circles

(b) $9x^2 + y^2 - 2y - z^2 + 2z = 0$

or $9x^2 + (y-1)^2 - (z-1)^2 = 0$



It is a hyperpoloid

$z=1, \quad 9x^2+(y-1)^2=0$

$z=0 \quad 9x^2+(y-1)^2=1$

$z=k \quad 9x^2+(y-1)^2=(k-1)^2$

Ellipses

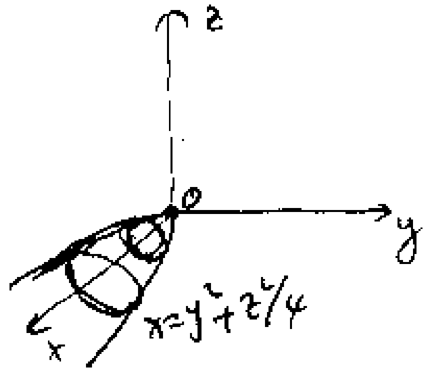
3. (a) A line, A point $(0, 8, -2)$,

The direction $\langle 2, 1, 0 \rangle$

(b). A plane A point $(0, 0, -1/4)$

The normal direction, $\vec{n} = \langle 1, -2, 4 \rangle$

(c) An elliptic paraboloid



$x=0, \quad 0=y^2+z^2/4$

$x=1 \quad 1=y^2+z^2$

(4) $x^2+x+y^2-3y+z^2=a$

$(x+\frac{1}{2})^2+(y-\frac{3}{2})^2+z^2=a+\frac{1}{4}+\frac{9}{4}$

So if $a+\frac{1}{4}+\frac{9}{4}=a+\frac{10}{4}=a+\frac{5}{2}\geq 0$, The equation is a sphere. The center is $(-\frac{1}{2}, \frac{3}{2}, 0)$ and the radius is $\sqrt{a+\frac{5}{2}}$.