

Solution to Quiz 1

1 (5) (a) $|BC| = \sqrt{(1-1)^2 + (0+1)^2 + (-2)^2} = \sqrt{5}$

(b) $\vec{BA} = \langle 1, 0, 1 \rangle$, $\vec{BC} = \langle 0, 1, -2 \rangle$

(c) $\vec{BA} \cdot \vec{BC} = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot (-2) = -2$

$$\cos \theta = \frac{-2}{\sqrt{2} \sqrt{5}} = \frac{-2}{\sqrt{10}} = -\frac{1}{\sqrt{10}}$$

(d) $\vec{BA} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{vmatrix} = \vec{i} \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$$= -\vec{i} + 2\vec{j} + \vec{k}$$

(3) Area = $|\vec{BA} \times \vec{BC}| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$

(2) A, B, and C are not in a straight line.

(5) (e) $(x-1)^2 + (y+1)^2 + z^2 = 2.5^2$

2 (5) (a) $3(x-1) - (y+1) + 4(z-2) = 0$ or $3x - y + 4z - 12 = 0$

(5) (b) $D = \frac{|3x_1 - 2x_0 + (-1) - 2|}{\sqrt{3^2 + (-2)^2 + 1^2}} = \frac{|2-2|}{\sqrt{14}} = 0$

(5) (c) $\begin{cases} x = 1 + 3t \\ y = -1 - t \\ z = 2 + 4t \end{cases}$ or $\vec{r} = \langle 1, -1, 2 \rangle + \langle 3, -1, 4 \rangle t$
 t is the parameter

3. (a) It's a line. A point: $(-1, 0, 5)$

(5) The direction $\langle -1, 0, 10 \rangle$

(5) (b) A plane. A point $(0, 0, 1)$
 The direction $\langle 2, -1, -1 \rangle$

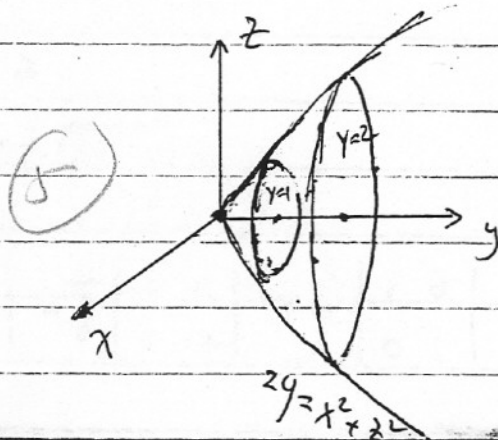
(c) (5) It's an elliptic paraboloid

$$y=0 \Rightarrow x^2+z^2=0$$

$$y=\frac{1}{2} \Rightarrow x^2+z^2=1$$

$$y=2 \Rightarrow x^2+z^2=4$$

...



4 Use complete squares

$$(5) (x+2)^2 + (y-2)^2 + z^2 = a + b$$

(5) If $a + b \geq 0$ or $a > -b$, the equation is a sphere. The center is

(5) $(-2, 2, 0)$, The radius is $\sqrt{a+b}$

$$5. \vec{v}_1 = \langle 1, -1, 3 \rangle, \vec{v}_2 = \langle -1, 2, 1 \rangle$$

$$\vec{v}_1 \times \vec{v}_2 = -7\vec{i} - 4\vec{j} + \vec{k}$$

Take a point on L_1 : $(1, 2, 0)$, The equation of the plane passing through L_1 and the point is

$$-7(x-1) - 4(y-2) + (z-0) = 0$$

Note that this plane is parallel to L_2 ,

Therefore, the distance from any point on L_2 to the plane is the answer, see the diagram.

$$D = \frac{|-7(2-1) - 4(1-2) + (4-0)|}{\sqrt{49+16+1}} = \frac{1}{\sqrt{66}}$$

