Examples shown that IIM produces second order accurate solution and the first order derivatives in 1D

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1 Introduction

Here are some sample computations demonstrating the second order accurate derivative at the interface for the 1-D interface problem presented in "The Immersed Interface Method" by Dr.Li by using a two sided method directly related to the IIM. The 1-D Interface problem where $x \in (0, \alpha) \cup (\alpha, 1)$ is often expressed as:

$$(\beta_x u)_x - \sigma u = f$$

In the examples presented here, all the functions have finite jumps at the interface α. The same β and σ are used throughout, and the proper Dirichlet boundary condition according the the actual solution are always chosen. Note

$$\beta(x) = \begin{cases} 1 + \dfrac{\cos(x)}{2} & \text{if} \quad x \leq \alpha \\ 1 + x^2 & \text{if} \quad x > \alpha \end{cases}$$

and

$$\sigma(x) = \begin{cases} (1+x)^2 & \text{if} \quad x \leq \alpha \\ \ln(2+x) & \text{if} \quad x > \alpha \end{cases}$$

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2 Example 1

Consider the problem where

$$f(x) = \begin{cases} (120)^2(-(\dfrac{\cos(x)}{2}+1))\sin(120x) - \dfrac{120}{2}\sin(x)\cos(120x) - (1+x)^2\cos(120x) & \text{if} \quad x < \alpha \\ (30)(-30(x^2+1)\cos(30x) - 2x\sin(30x)) - \ln(2+x)\cos(30x) & \text{if} \quad x < \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(x) = \begin{cases} \sin(120x) & \text{if} \quad x \leq \alpha \\ \cos(30x) & \text{if} \quad x > \alpha \end{cases}$$

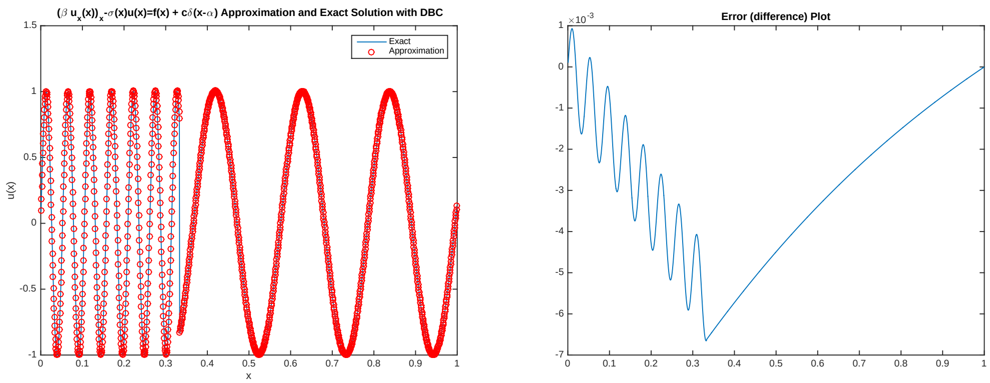

Figure 1: (a): An example of a computed solution with $n = 1280$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. (b): The error difference of the solution and actual solution at the grid points.

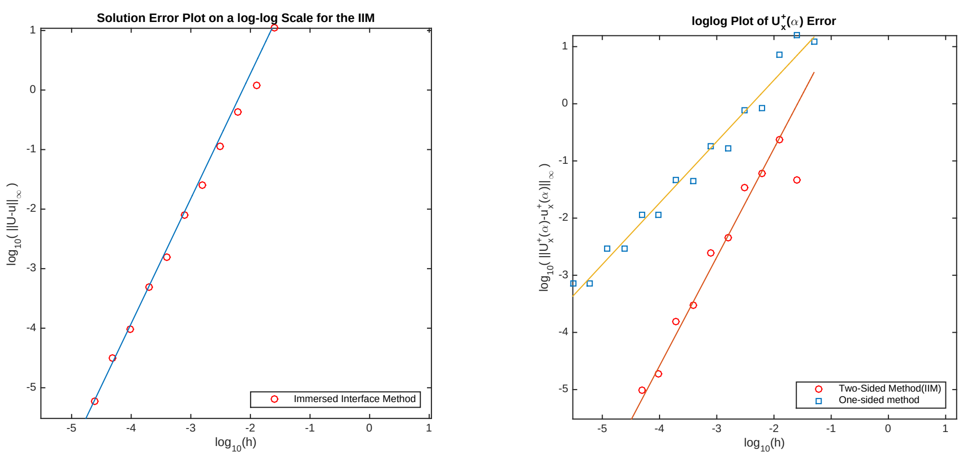

Figure 2: (a): Grid refinement analysis of the approximate solution with 15 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 2.1 (b):Grid refinement analysis of the right derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is 1.08 and 1.90 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

3 Example 2

Consider the problem where

$$f(x) = \begin{cases} (120)^2(-(\frac{\cos(x)}{2} + 1))\sin(120x) - \frac{120}{2}\sin(x)\cos(120x) - (1+x)^2\sin(120x) & \text{if} \quad x \leq \alpha \\ \frac{x^2+200x-1}{(x+100)^2} - \ln(2+x)\ln(100+x) & \text{if} \quad x > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(x) = \begin{cases} \sin(120x) & \text{if} \quad x \le \alpha \\ \ln(100 + x) & \text{if} \quad x > \alpha \end{cases}$$

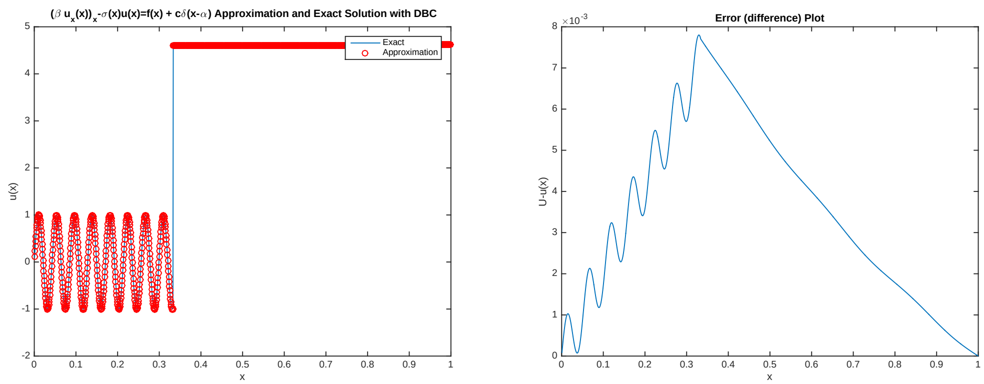

Figure 3: (a): An example of a computed solution with $n = 1280$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

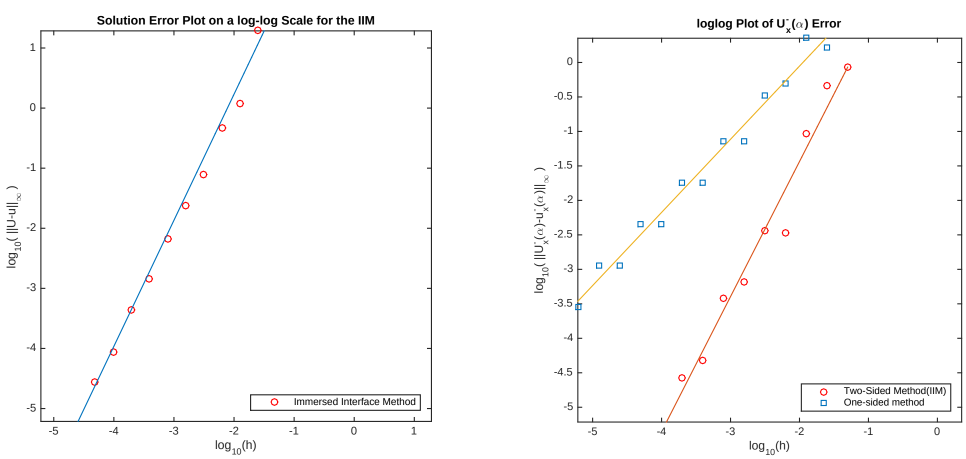

Figure 4: (a): Grid refinement analysis of the approximate solution with 14 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 2.1 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 1.06 and 1.96 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

4 Example 3

Consider the problem where

$$f(x) = \begin{cases} (15)^2(1+\dfrac{\cos(x)}{2})\sinh(15x) - (15)\dfrac{\sin(x)}{2}\cosh(15x) - (1+x)^2\sinh(15x) & \text{if} \quad x \le \alpha \\ (-(x^2+1)\cosh(x) - 2x\sinh(x)) - \ln(2+x)\cosh(x) & \text{if} \quad x > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(x) = \begin{cases} \sinh(15x) & \text{if} \quad x \le \alpha \\ \cosh(x) & \text{if} \quad x > \alpha \end{cases}$$

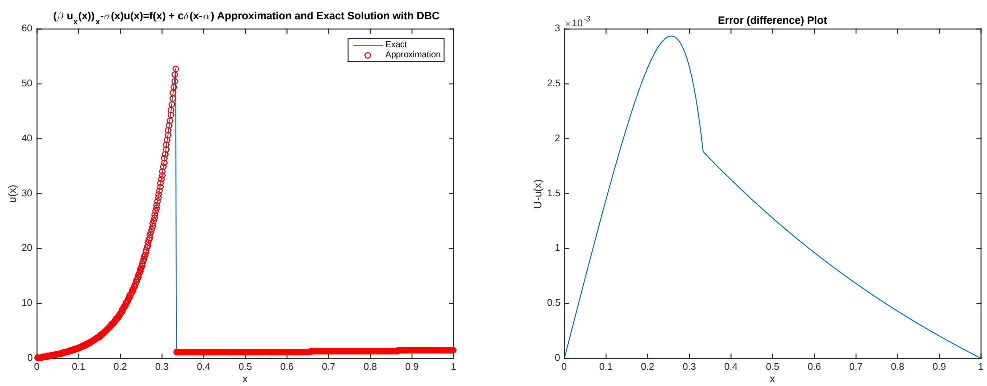

Figure 5: (a): An example of a computed solution with $n = 640$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. (b): The error difference of the solution and actual solution at the grid points.

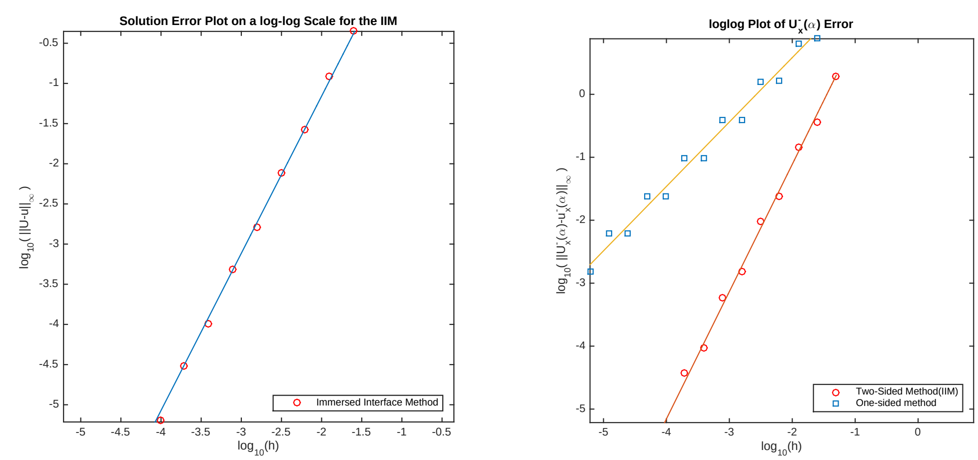

Figure 6: (a): Grid refinement analysis of the approximate solution with 14 refinements starting at $n = 20$.Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 1.96 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 1.02 and 2.02 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

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