Examples of Second Order Accurate Derivatives Using the IIM in Spherical Coordinates with Axis Symmetry

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The axis-symmetric interface problem in spherical coordinates where $r \in (0, \alpha) \cup (\alpha, 1)$ can be expressed as: $r \in (0, \alpha) \cup (\alpha, 1)$ is often expressed as:

$$\frac{1}{r^2}(r^2 \beta u_r)_r = f$$

In the examples presented here, β, u_r, u have finite jumps at the interface α. The same β is used throughout, and the proper Dirichlet boundary condition according the the actual solution are always chosen. Note that

$$\beta(x) = \begin{cases} 1 \quad \text{if} \quad r \leq \alpha \\ 100 \quad \text{if} \quad r > \alpha \end{cases}$$

1 Example 1

Consider the problem where

$$f(r) = \begin{cases} \frac{-(2\sin(r)}{r} - \cos(r) \quad \text{if} \quad r \leq \alpha \\ (\frac{2}{r} - r)\cos(r) - 4\sin(r) \quad \text{if} \quad r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \cos(r) \quad \text{if} \quad r \leq \alpha \\ r\cos(r) \quad \text{if} \quad r > \alpha \end{cases}$$

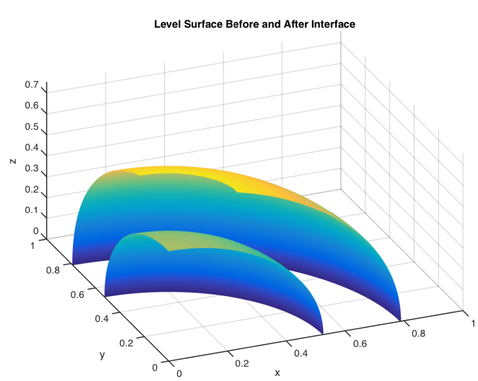

Figure 1: (a): The surface of the approximation .

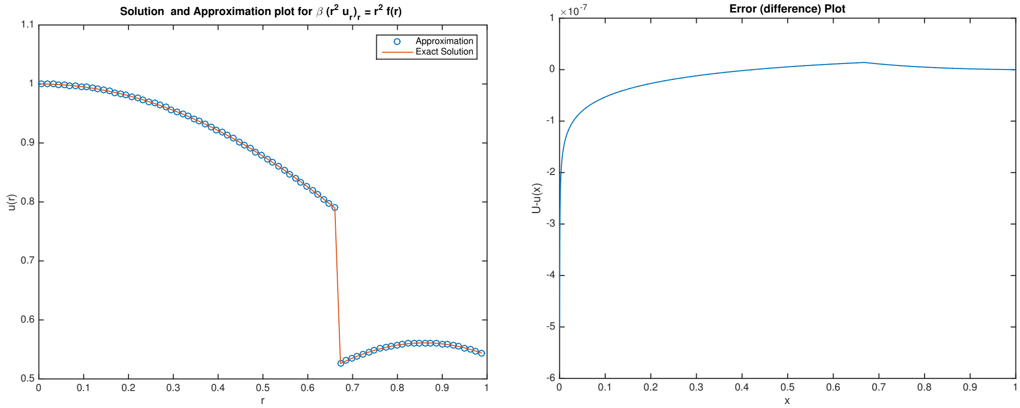

Figure 2: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

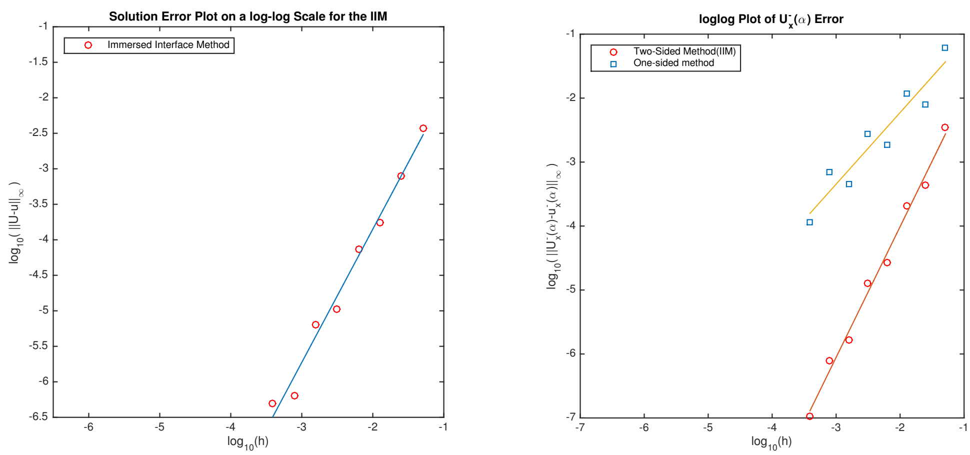

Figure 3: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 1.9 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.96 and 1.92 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

2 Example 2

Consider the problem where

$$f(r) = \begin{cases} -3r(4\sin(r^3) + 3r^3\cos(r^3) & \text{if} \quad r \le \alpha \\ 6\cos(r^2) - 4r^2\sin(r^2) & \text{if} \quad r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \cos(r^3) & \text{if} \quad r \le \alpha \\ \sin(r^2) & \text{if} \quad r > \alpha \end{cases}$$

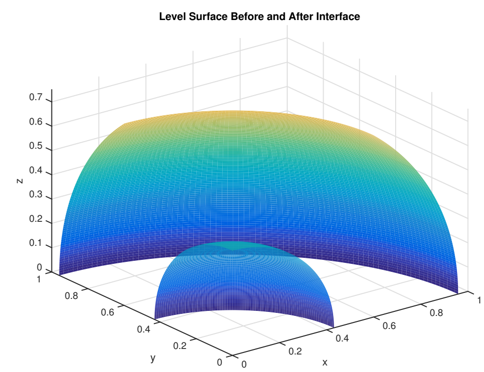

Figure 4: (a): The surface of the approximation .

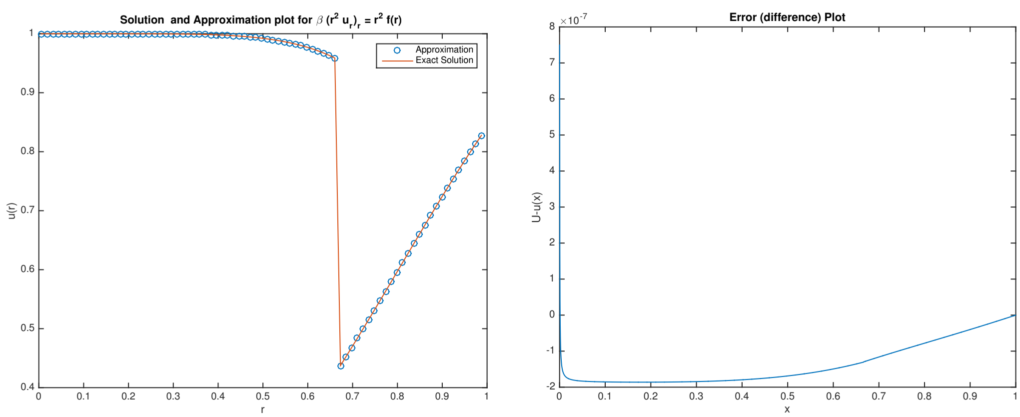

Figure 5: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

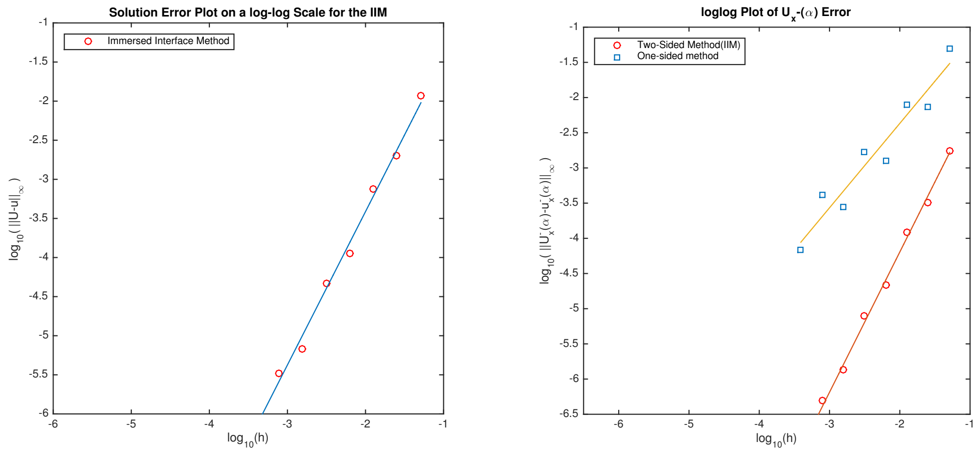

Figure 6: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 1.96 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.99 and 1.89 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

3 Example 3

Consider the problem where

$$f(r) = \begin{cases} \frac{-2(\sin(2r)+r\cos(2r))}{r} & \text{if} \quad r \leq \alpha \\ \frac{2(\sin(2r)+r\cos(2r))}{r} & \text{if} \quad r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \cos^2(r) & \text{if} \quad r \leq \alpha \\ \sin^2(r) & \text{if} \quad r > \alpha \end{cases}$$

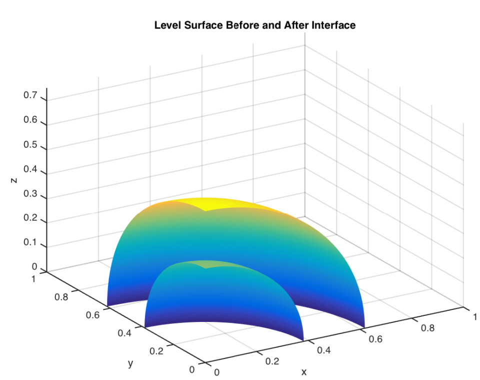

Figure 7: (a): The surface of the approximation .

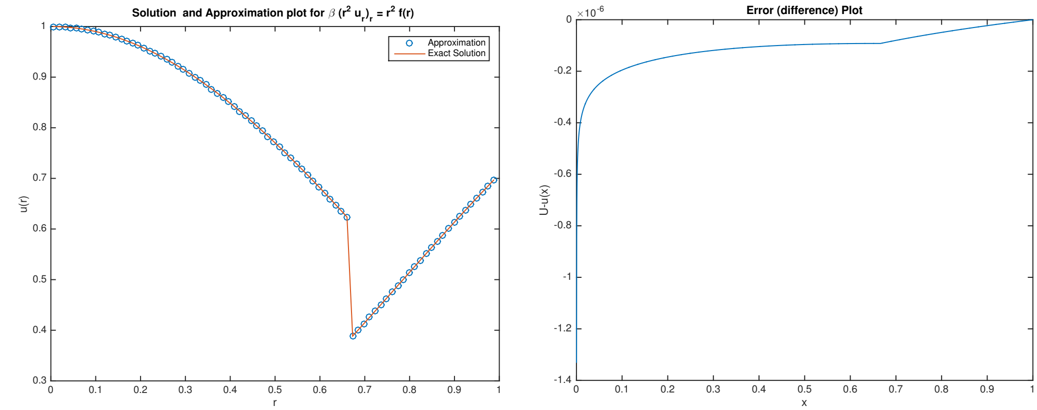

Figure 8: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

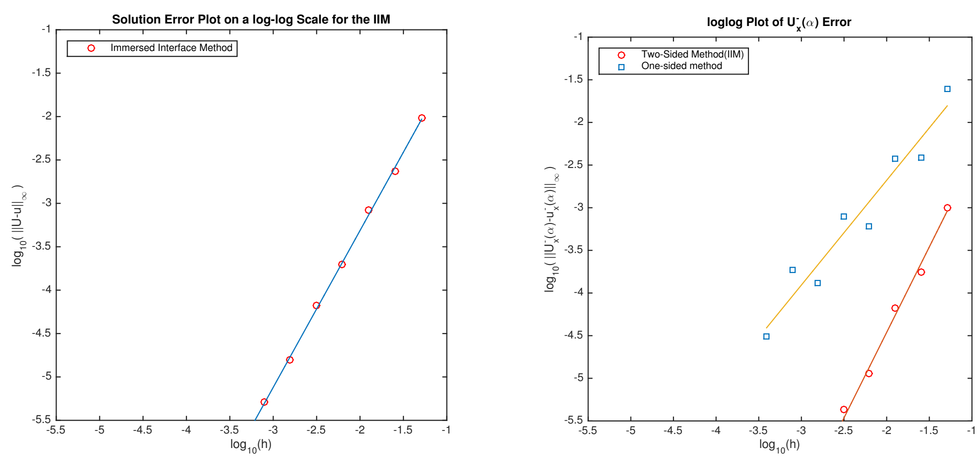

Figure 9: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 1.8 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.94 and 1.94 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

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