

Examples of Second Order Accurate Derivatives Using the IIM in Polar Coordinates with Axis Symmetry

Nnenna Anoruo, Kevin Luna, William Reese, Justin Rivera
 Research Mentor: Dr.Zhilin Li

NC State University REU 2015

The axis-symmetric interface problem in polar coordinates where $r \in (0, \alpha) \cup (\alpha, 1)$ can be expressed as:

$$\frac{1}{r}(r\beta u_r)_r = f$$

In the examples presented here, β , u_r , u have finite jumps at the interface α . The same β is used throughout, and the proper Dirichlet boundary condition according the the actual solution are always chosen. Note that

$$\beta = \begin{cases} 1 & \text{if } r \leq \alpha \\ 100 & \text{if } r > \alpha \end{cases}$$

1 Example 1

Consider the problem where

$$f(r) = \begin{cases} \frac{-(\sin(r)+r \cos(r))}{r} & \text{if } r \leq \alpha \\ \frac{9 \cos(9r)}{r} - 81 \sin(9r) & \text{if } r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \cos(r) & \text{if } r \leq \alpha \\ \sin(2r) & \text{if } r > \alpha \end{cases}$$

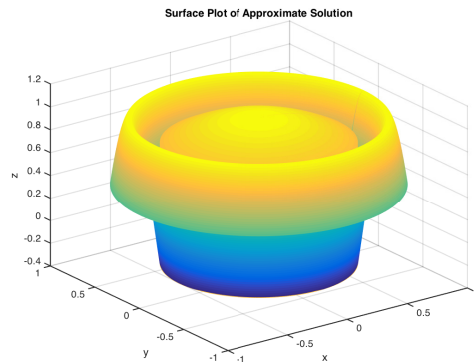


Figure 1: (a): The surface of the approximation .

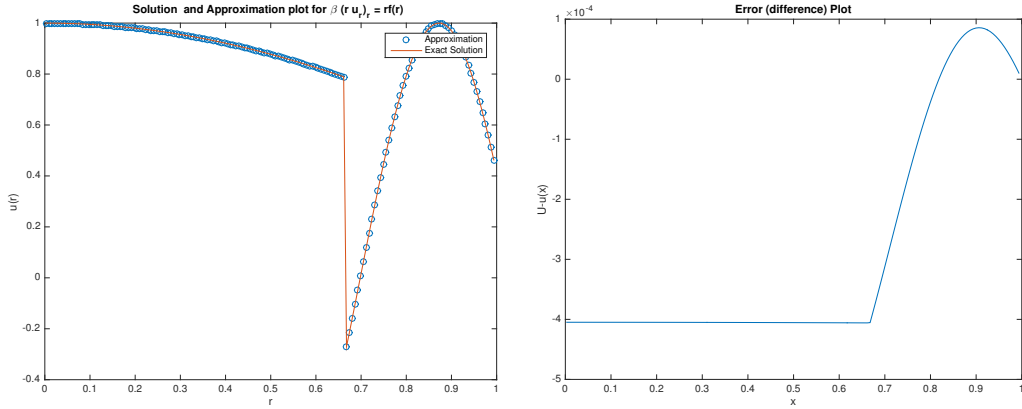


Figure 2: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

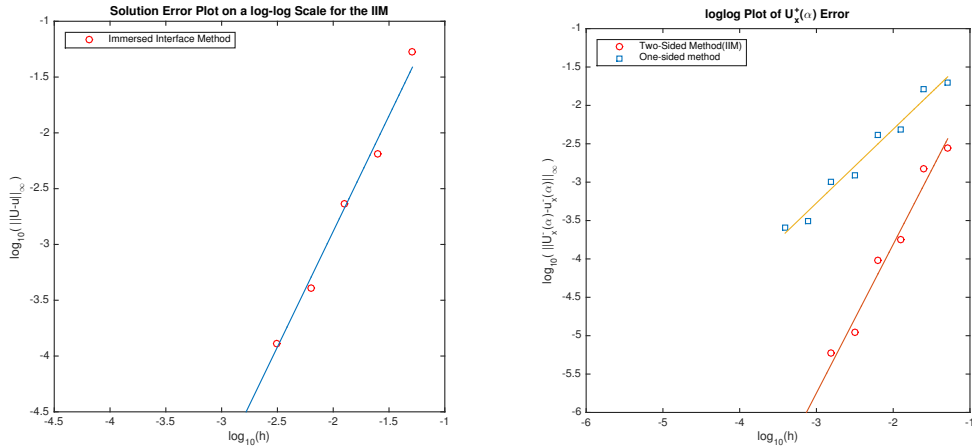


Figure 3: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 2.07 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.96 and 1.94 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

2 Example 2

Consider the problem where

$$f(r) = \begin{cases} \frac{-2(\sin(2r)+2r \cos(2r))}{r} & \text{if } r \leq \alpha \\ \frac{-r^2 \sin(r)+\sin(r)-r \cos(r)}{r^3} & \text{if } r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \cos(2r) & \text{if } r \leq \alpha \\ \frac{\sin(r)}{r} & \text{if } r > \alpha \end{cases}$$

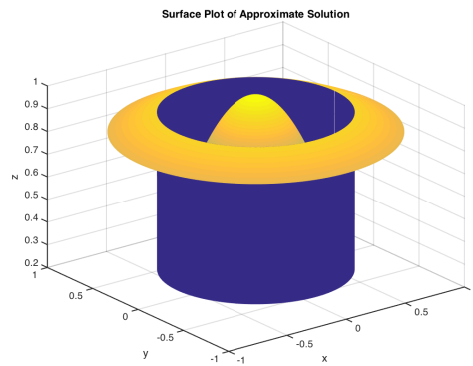


Figure 4: (a): The surface of the approximation .

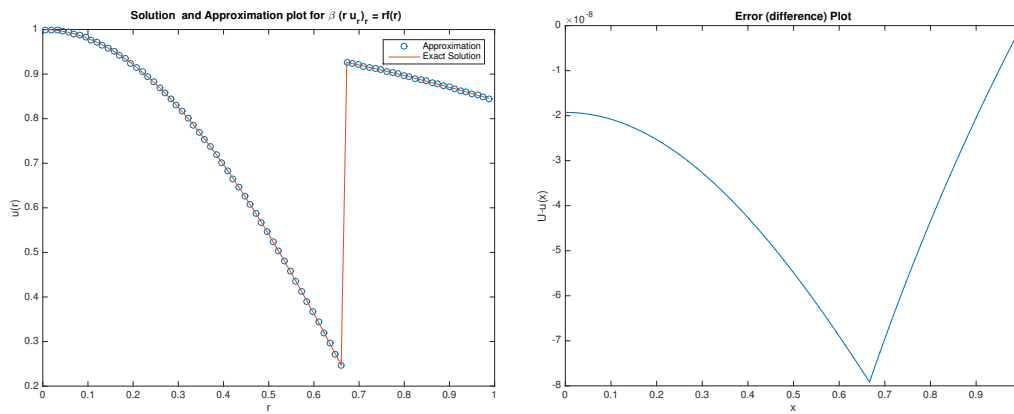


Figure 5: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

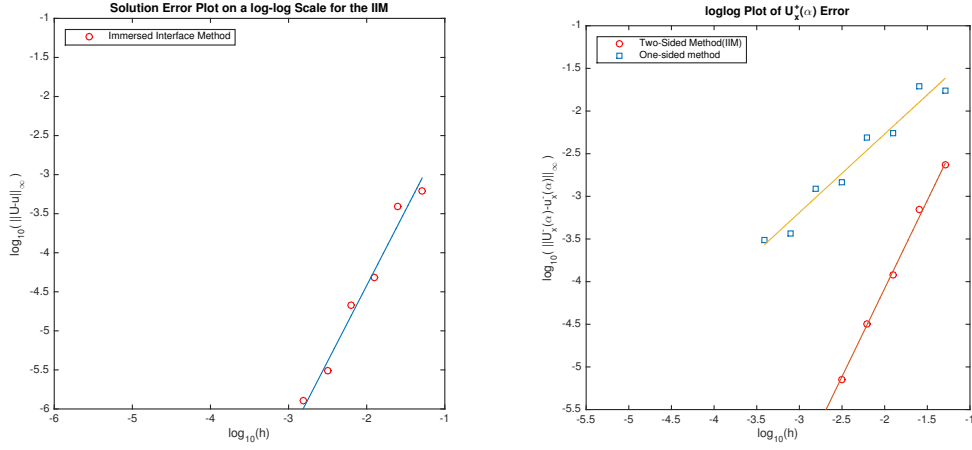


Figure 6: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 1.99 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.92 and 2.07 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

3 Example 3

Consider the problem where

$$f(r) = \begin{cases} \frac{2}{r(2+r)^2} & \text{if } r \leq \alpha \\ \frac{-(\sin(r)+r \cos(r))}{r} & \text{if } r > \alpha. \end{cases}$$

The exact solution can be shown to be

$$u(r) = \begin{cases} \ln(2+r) & \text{if } r \leq \alpha \\ \cos(r) & \text{if } r > \alpha \end{cases}$$

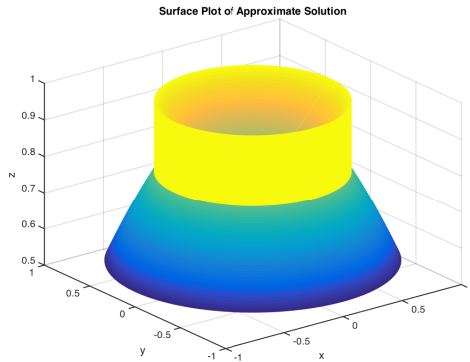


Figure 7: (a): The surface of the approximation .

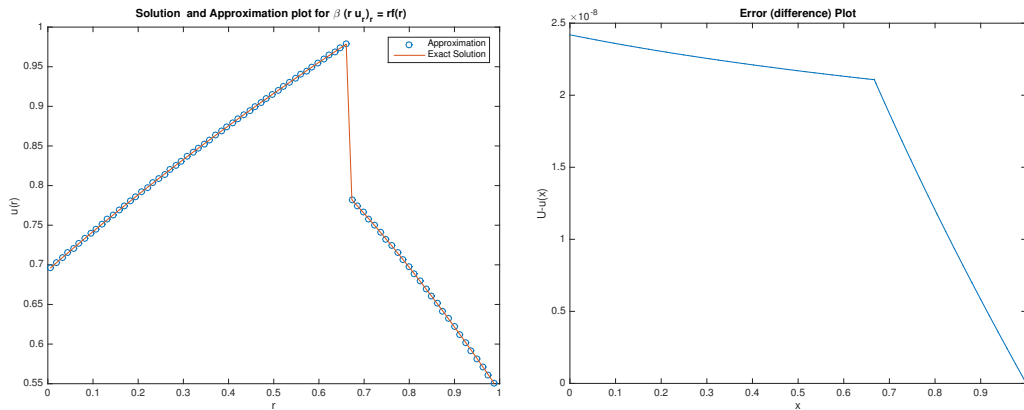


Figure 8: (a): An example of a computed solution with $n = 160$ grid divisions. Note the actual solution has its discontinuity connected by the plotter. Note that the axis has been adjusted so some points may not be visible. (b): The error difference of the solution and actual solution at the grid points.

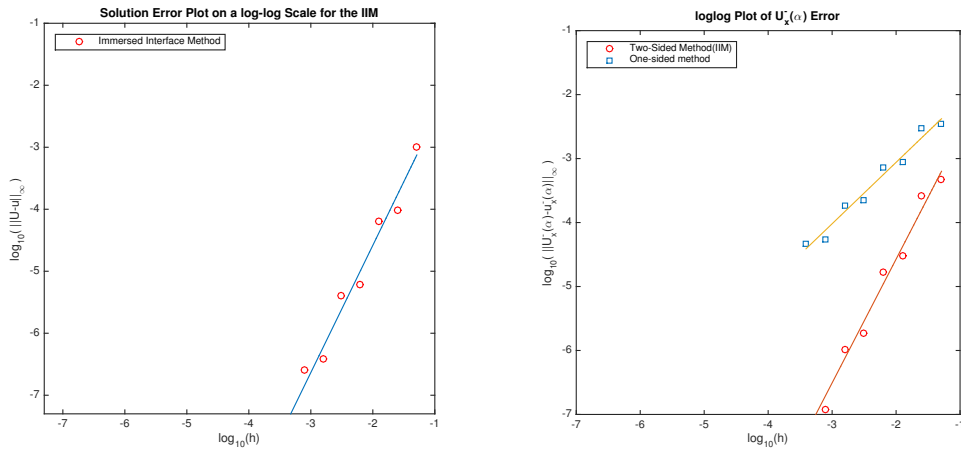


Figure 9: (a): Grid refinement analysis of the approximate solution with 8 refinements starting at $n = 20$. Note that the axis has been adjusted so some points may not be visible. The slope of the regression line fit is 2.05 (b):Grid refinement analysis of the left derivative at the interface with the one-sided and two-sided(IIM) methods. Note that a linear regression fit shows that the slope of the line of best fit is about 0.96 and 1.93 for the one-sided and two-sided methods respectively. This shows the IIM two sided method gave an order 2 accurate approximation while standard one-sided methods gave a first order accurate approximation.

4 Acknowledgments

This project was funded by the following agencies and grants:
 NSF DMS-1461148
 NSA H98230-15-1-0024